Jour. Ind. Soc. Ag. Statistics
43 (1), 1991 : 59 – 70

ON MINIMUM VARIANCE LINEAR UNBIASED ESTIMATION IN T-CLASSES

N.K. Bhargava[1], B.D. Tikkiwal[2]
and
G.C. Tikkiwal[3]
(Received : February, 1989)

Summary

Using partition of sample space due to Tikkiwal (1972); Bhargava and Tikkiwal (1978) re-defined T_5 and T_6 - classes of linear estimators, so as to make the seven classes of linear estimators broad-based. This paper discusses the existence or otherwise of minimum-variance-linear-unbiased-estimator (MVLUE) in the T_5 and T_6-classes and in the most general class T_7. It also gives a unified proof for the non-existence of MVLUE in the T_3 - class of Prabhu-Ajgaonkar and Tikkiwal (1961) for sampling schemes with or without replacement.

Keywords : T-classes, partition of sample space, Minimum- variance linear-unbiased-estimator (MVLUE), Unified approach to sampling with and without replacement, Basu's estimator. Midzuno sampling scheme, Horvitz-Thompson sampling scheme.

Introduction

A review of seven T-classes of linear estimators, starting with the work of Horvitz and Thompson [4] is given earlier by Bhargava and Tikkiwal [2]. While reviewing the literature, they re-defined T_5 and T_6-classes using partition of sample space (Tikkiwal, [9]), so as to make the seven classes broad-based. It is shown by them [Theorem 3.1, part (b), p. 15] that Godambe's general class [3] is a sub-class of T_7-class. Thus, Godambe showed only the non-existence of the minimum-variance-linear-unbiased-estimator (MVLUE) in this sub-class. This paper, therefore, discusses the existence or otherwise of MALUE in the new T_5 and T_6-classes and also in the broad T_7- class. Further, Prabhu-Ajgaonkar and Tikkiwal [8] defined T_3-class for sampling with or without replacement differently (Table 3.1, p. 18) from other authors (Godambe [3] ; Koop [5] [6]) and proved the non-existence of MVLUE separately for sampling schemes with replacement and

[1] Planning Office, Bank of Baroda, Jaipur
[2] U.G.C. Emeritus Fellow, B-45 Dev Nagar, Tonk Road, Jaipur
[3] University of Jodhpur, Jodhpur.

for those without replacement. This paper gives a unified proof for the non-existence of MVLUE in this T_3-class of Prabhu-Ajgaonkar and Tikkiwal for sampling schemes with or without replacement.

It is shown that T_5 and T_7-classes of linear unbiased estimators are, in general, non-empty. However, there is no MVLUE in these classes.

The new T_6-class of linear unbiased estimators is non-empty for simple random sampling without replacement and the classical estimator, the sample mean, is MVLUE in this class. However, for simple random sampling scheme with replacement, Midzuno sampling scheme and Horvitz-Thompson's scheme, this class is empty in general. The estimator for such sampling schemes lies in another new class due to Tikkiwal, [11] Which depends on a particular S_p, the draw and the out-come at that draw.

For simple random sampling with replacement and varying probabilities with replacement, the T_6-class may still be treated as empty for search of estimators for these sampling schemes; as in these cases the estimator also lies in still another new class due to Tikkiwal [11] which depends only on an S_p for all samples in that S_p.

2. *Approach to the Problem and Notations Used*

Let $u_1, u_2, \ldots, u_N$ be the N distinct units of given finite population. Let x_i for $i = 1, 2, \ldots, N$ denote the observation on the i^{th} units u_i of the population for the character X under study. Because of what is said in Section 1, the problem is to examine existence or otherwise of MVLUE in T_5, T_6 and T_7-classes, of population total $T(X) = \sum_{i=1}^{N} x_i$ based on the observations x_i on units occuring in a given sample s_t, of size n drawn from the finite population according to a certain sampling scheme in which units are drawn one by one with or without replacement. In addition to solve this problem, we are to provide a unified proof for the non-existence of MVLUE in the T_3-class due to Prabhu- Ajgaonkar and Tikkiwal for sampling with or without replacement.

We observe here that every sampling scheme gives rise to probability $p(s_t)$ to be associated with the sample s_t such that $\Sigma\, p(s_t)=1$, the summation being over all t, the number of possible samples according to the given sampling scheme. If the units are drawn one by one all along without replacement, the number of possible samples is $\binom{N}{n} n\,!$, where as for unit by unit draw all along with replacement, this number is N^n. In the latter case none of $p(s_t)$ is zero; however in the former case we have to take $p(s_t)=0$ for those $N^n-\binom{N}{n} n\,!$ samples which do not occur, if we regard the number of possible samples still to be N^n. Thus, by taking relevant $p(s_t)$ to be zero, we can always take the number of possible samples to be

N^n in any unit-by-unit draw sampling scheme with or without replacement or partially with replacement and partially without replacement. This we do have in order to deal with the problem of determining MVLUE simultaneously for all such schemes. In this sense our approach is unified one, as one does not have to develop theory for sampling without replacement separately from that for sampling with replacement which is generally done otherwise as stated in Section 1. It may by noted here that it is easy to invent sampling schemes which give rise to more than N^n possible samples. We rule out such sampling schemes from our consideration.

For wider applicability of the results, we examine first the existence of unbiased estimators in each of the four classes for $X=(x_1, x_2, \ldots, x_N)$, when x_i can take all values in the open interval $(-\infty, \infty)$. If we regard X as a vector lying in the N-dimensional vector space $R^{(N)}$, then it amounts to examining the existence of unbiased estimators in each of the four classes for all $X \in R^{(N)}$. If there is no unbiased estimator in a given class, then that class is said to be empty. When a particular class in non- empty, we then proceed to examine for the existence or otherwise of MVLUE in that class again for all $X \in R^{(N)}$.

Before discussing the main problem, we present below the partition of the sample space, as referred in Section 1, alongwith the necessary terminology and notations to be used subsequently.

Let the sample s_t consist of k distinct units $(i_1, i_2, \ldots, i_k)$ where k=1,2,...,n. Let i_j-th unit in this sample for j=1,2,...,k occur $R^{(p)}_{i_j}$ times with $\sum j\, R^{(p)}_{i_j} = n$. Let $R^{(p)} = \left(R^{(p)}_{i_1}, R^{(p)}_{i_2}, \ldots, R^{(p)}_{i_k}\right)$. Let M_{k1} denote the total number of $R^{(p)}$ vectors, the number which is independent of the nature of k units and is equal to $\binom{n-1}{k-1}$ [Koop, [7], p.20]. For a given $R^{(p)}$, we have $M_{klp} = [n! \big/ \pi_{j=1}^{k} R^{(p)}_{i_j}!]$ possible number of samples for different ordering of the same units. Thus, the number of possible samples for a given vector R(p) is also independent of the nature of k units. Let S_p be the set of these M_{klp} samples; each sample having the same k units $(i_1,i_2,\ldots,i_k)$ with i_j-th units occuring $R^{(p)}_{i_j}$ times for $j = 1, 2, \ldots, k$. For a given k distinct units, we have M_{kl} such sets. Let C_1 denote the class of these M_{kl} sets for i=1,2,..., $\binom{N}{k}$: $\binom{N}{k}$ being the number of ways in which k distinct units can be chosen out or N units. Let $p(S_p)$ and $p(C_1)$ denote respectively the probabilities of getting a particular $S_p \in C_1$ and of getting that C_1 itself. Let $p(s_t)$, as above, denote the probability of a sample $s_t \in S_p$. Let $\Sigma_{(1)}$ stand for the summation over all possible samples s_t in different S_p's spread over different C_1's and $\Sigma_{(i)}$ for summation over all samples s_t containing i-th unit for summation over all samples s_t containing i-th for i=1,2,..,N.

3. *On the Non-Existence of MVLUE in T_3-Class*

An estimator in T_3-class, based on the sample s_t for $t = 1, 2, \ldots, N^n$ is given by

$$\hat{T}_3 = \beta^{s_t}\left(\sum_{j=1}^{k} R_{i_j}^{(p)} \cdot x_{i_j}\right) \tag{3.1}$$

where β^{S_t} is the weight dependent on the sample s_t. We first determine the weights β^{S_t}, in number N^n, such that $E(\hat{T}_3) = T$ for all $x \in R^{(N)}$ and thereby show that this class of linear unbiased estimators is non-empty. Here

$$E(\hat{T}_3) = \sum_{i=1}^{N} x_i \left[\Sigma_{(i)}\, \beta^{S_t} R_i^{(p)}\, p(s_t)\right]$$

which in turn gives

$$\Sigma_{(i)}\, \beta^{S_t} R_i^{(p)}\, p(s_t) = 1 \tag{3.2}$$

for $i = 1, 2, \ldots, N$ as the necessary and sufficient conditions of unbiasedness. These N equations are consistent as

$$\beta^{S_t} = 1 / [p(s_t)\, C] \text{ for all } s_t \text{ with } p(s_t) \neq 0\,;\ \beta^{S_t} = \beta_0\,,$$

an arbitrary constant for all s_t with $p(s_t) = O$; and with

$$C = [\Sigma_{(i)}\, R_i^{(p)}] = \sum_{i=1}^{n} \sum_{l=1}^{\binom{N-1}{k-1}} \sum_{p=1}^{M_{kl}} R_i^{(p)} M_{klp}$$

provide a solution of (3.2). The quantity C, itself, means the total number of times i-th unit occurs in different samples. Thus, for sampling without replacement,

$$C = \binom{N-1}{n-1}\, n\,!$$

and for sampling with replacement

$$C = \sum_{j=1}^{n} j \binom{n}{j} (N-1)^{n-j}$$

$$= N^n \left[\sum_{j=1}^{n} j \binom{n}{j} \left(\frac{1}{N}\right)^{j} \left(1 - \frac{1}{N}\right)^{n-j} \right]$$

$$= n\, N^{n-1}$$

Thus, the quantity C is independent of i and thus β^{S_t} varies only with the probability of the sample.

Thus, T_3-class is, in general, a non-empty class. The variance of the estimator $\hat{T}_3$ is given by

$$V(\hat{T}_3) = \Sigma_{(1)} \left[\beta^{S_t} \left(\Sigma_j R_{ij}^{(p)} x_{i_j} \right) \right]^2 p(s_t) - T^2 \tag{3.3}$$

For minimisation of the variance of T_3 with respect to the weights β^{S_t}, we take

$$\varnothing = V(\hat{T}_3) - 2 \sum_i \lambda_i \left[\Sigma_{(i)} \beta^{S_t} p(s_t) R_i^{(p)} - 1 \right] \tag{3.4}$$

where λ_i is the Lagrange's undetermined multiplier corresponding to (3.2) for $i = 1, 2, \ldots, N$. Equating $\partial\varnothing / \partial\beta^{S_t}$ to zero, we get

$$\beta^{S_t} = \frac{\left[\sum_j \lambda_{i_j} R_{ij}^{(p)} \right]}{\left[\sum_j R_{ij}^{(p)} x_{i_j} \right]^2}, \text{ for all } s_t \text{ with } p(s_t) \neq 0 \tag{3.5}$$

The choice of β^{S_t} should hold for all $x \in R^{(N)}$ in order that the variance of $\hat{T}_3$ be minimum for all $x \in R^{(N)}$. Let us take a particular population $X = (0, 0, \ldots ; x_i \neq 0, 0, \ldots, 0)$ with unit $u_i \in s_t$. Then (3.5) implies that

$$\beta^{S_t} = \frac{\left[\sum_j \lambda_{i_j} R_{ij}^{(p)} \right]}{\left(R_i^{(p)}\right)^2 x_i^2}, \text{ for all } s_t \text{ with } p(s_t) \neq 0 \tag{3.6}$$

Let $X=(x_i, x_2, \ldots, x_N)$ with all x_i in the given s_t being zero. Then (3.5) implies that $\Sigma_j \left(\lambda_{i_j} R_{i_j}^{(p)}\right) = 0$. So $\beta^{S_t} = 0$, for all s_t with $p(s_t) \neq 0$. Similarly other $\beta^{S_{t'}}$'s for such s_t's can be shown to be zero. But this choice of β^{S_t} does not satisfy the conditions of unbiasedness and hence there is a contradiction, showing there by the non-existence of MVLUE. This gives the unified proof of the non-existence of MVLUE in T_3-class under consideration for sampling schemes with or without replacement.

4. *On the Non-existence of MVLUE in T_5-Class*

An estimator in T_5-class, based on a sample $s_t \in S_p$ is defined as

$$\hat{T}_5 = \sum_j \beta_{i_j}^{S_p} R_{i_j}^{(p)} x_{i_j} \tag{4.1}$$

It has been shown by Bhargava and Tikkiwal [2] that T_2-class estimator due to Godambe [3] and Koop [5] ; and T_3 and T_5-class estimators due to Koop [5] lie in this class. Basu's estimator [1] based on distinct units for simple random sampling with replacement also lies in this class. Thus it is a wider class and is non-empty.

We now give a general proof of this class being non-empty class of linear unbiased estimators. For this type of estimator to be unbiased, we must have

$$\sum_{k=1}^{n} \sum_{l=1}^{\binom{N-1}{k-1}} \sum_{p=1}^{M_{kl}} \left[p(S_p)\, R_i^{(p)} \beta_i^{S_p}\right] = 1 \tag{4.2}$$

for $i = 1, 2, \ldots, N$. A consistant solution of these N equations is given by

$\beta_i^{S_p} = 1/\left[p(S_p) C R_i^{(p)}\right]$ for all such weights with

$$C = \sum_{k=1}^{n} \sum_{l=1}^{\binom{N-1}{k-1}} \sum_{p=1}^{M_{kl}} (1) = \sum_{k=1}^{n} \binom{N-1}{k-1} M_{kl} = \sum_{k=1}^{n} \binom{N-1}{k-1} \binom{n-1}{k-1}$$

showing that the T_5-class, in general, is non-empty class. That this estimator is different from Basu's estimator for SRSWR is easily noted. The variance of the estimator $\hat{T}_5$ is given by

$$V(\hat{T}_5) = \sum_k \sum_l \sum_p \left[\sum_j \beta_{i_j}^{S_p} R_{i_j}^{(p)} x_{i_j}\right]^2 p(S_p) - T^2 \tag{4.3}$$

By defining $\emptyset$ as in Section 3 with λ_i as Lagrange's undetermined multiplier corresponding to the i-th equation of (4.2) for i = 1, 2, . . . , N and then equating $\partial \emptyset / \partial \beta_i^{S_p}$ to zero, we have

$$\left[\sum_j \beta_{i_j}^{S_p} R_{i_j}^{(p)} x_{i_j}\right] x_i = \lambda_i \text{, for all } i \in s_t \in S_p \tag{4.4}$$

Considering the same two vector populations X, as in Section 3, in (4.4), we get $\beta_i^{S_p} = 0$ for all i and S_p. However, the set of $\beta_i^{S_p}$ with all their values zero, does not satisfy the conditions of unbiasedness, thereby bringing a contradiction. Thus MVLUE does not exist.

5. *On the Existence or Otherwise of MVLUE in T_6-Class*

5.1. General discussion. An estimator in T_6-class based on a particular sample s_t is given by

$$\hat{T}_6 = \sum_{r=1}^{n} \beta_r^{S_p} X_r \tag{5.1.1}$$

where $\beta_r^{S_p}$ denotes the weight to be associated with X_r, the variate value of the unit drawn at r-th draw. The total number of weights $\beta_r^{S_p}$ is

$$n\left[\sum_k \binom{N}{k} M_{kl}\right] = n\left[\sum_k \binom{N}{k}\binom{n-1}{k-1}\right].$$

The condition of unbiasedness gives rise to

$$\sum_{k=1}^{n} \sum_{l=1}^{\binom{N-1}{k-1}} \sum_{p=1}^{M_{kl}} \sum_{r=1}^{n} \left[\beta_r^{S_p}\left(\sum_{t=1}^{M_{klp}} p(s_t)\,\delta_{ri}\right)\right] = 1 \tag{5.1.2}$$

for i = 1, 2, . . . , N with

$$\delta_{ri} = \begin{cases} 1, & \text{if i–th unit occurs at r–th draw in sample } s_t \\ 0, & \text{otherwise} \end{cases}$$

A consistent solution of these N equations is given by

$$\beta_i^{S_p} = 1/\left[C.n \left(\sum_{t=1}^{M_{kp}} p(s_t)\,\delta_{ri} \right) \right] \tag{5.1.3}$$

for r and S_p with $C = \left[\sum_k \binom{N-1}{k-1} \binom{n-1}{k-1} \right]$, provided the expression for $\beta_r^{S_p}$ is same for all i. In such situations, T_6-class is a non-empty class, otherwise the estimator would jump to a new class due to Tikkiwal [11] which depends on a particular S_p, the draw and the out-come at that draw.

In order to examine the existence or otherwise of MVLUE for the situations where T_6-class is non-empty, we note that the variance of the estimator in T_6-class is given by

$$V(\hat{T}_6) = \Sigma_{(1)} \left[\left(\sum_r \beta_r^{S_p} X_r \right)^2 p(s_t) \right] - T^2 \tag{5.1.4}$$

After differentiating the function

$$\emptyset = V(\hat{T}_6) - 2 \sum_i \lambda_i \left[\Sigma_{(i)} \left(\left(\Sigma_r \beta_r^{S_p} \delta_{ri} \right) p(s_t) \right) - 1 \right]$$

with regard to a particular $\beta_r^{S_p}$ and putting that equal to zero, we get

$$\sum_{t=1}^{M_{kp}} \left[\left(\sum_r \beta_r^{S_p} X_r \right) X_{r'} \; p(s_t) \right]$$

$$= \sum_{i \in S_p} \lambda_i \left[\sum_{t=1}^{M_{kp}} \delta_{r'i}\, p(s_t) \right] \tag{5.1.5}$$

The MVLUE will now be determined by solving (5.1.5) for different $\beta_{r'}^{S_p}$ along with the conditions of unbiasedness given by (5.1.2). We now use these equations to obtain MVLUE in some special cases.

5.2 The minimum variance linear unbiased estimation in T_6-class for simple random sampling without replacement (SRSWOR)

From (5.1.3) we note that for SRSWOR, $C=\binom{N-1}{n-1}$ and so $\beta_r^{S_p} = N/n$ for all S_p and r. Thus, T_6-class is non-empty for this sampling scheme. In this case, (5.1.5) reduces to

$$\sum_{t=1}^{n!}\left[\left(\sum_r \beta_r^{S_p} X_r\right)X_{r'}\right]_{s_t} = \sum_{i\in S_p}\lambda_i\left[\sum_{t=1}^{n!}\delta_{r'i}\ )\right]_{s_t}$$

$$= (n-1)!\left[\sum_{i\in S_p}\lambda_i\right]$$

for different S_p and r'. For any given S_p, we have a set of n such equations. Taking any two such equations one for r' and other for r" (≠/r'), we have

$$\sum_{t=1}^{n!}\left[\left(\sum_r \beta_r^{S_p} X_r\right)X_{r'}\right]_{s_t} = \sum_{t=1}^{n!}\left[\left(\sum_r \beta_r^{S_p} X_r\right)X_{r''}\right]_{s_t}$$

which on further simplification gives

$$\left(\beta_{r'}^{S_p} - \beta_{r''}^{S_p}\right)\left[(n-1)\sum_{i\in s_t\in S_p} x_i^2 - \sum_{i,j\in s_t\in S_p} x_i x_j\right] = 0$$

Since, in general, the second factor is not equal to zero, therefore, $\left(\beta_{r'}^{S_p} - \beta_{r''}^{S_p}\right) = 0$. As it is true for any arbitrary r', r" and S_p ; $\beta_r^{S_p} = K$ for all S_p and r. That K=N/n is now seen from (5.1.2). Thus the classical estimator is MVLUE in this class for SRSWOR.

That, T_6-class is an empty class for simple random sampling with replacement for sample size n > 2 is seen by noting that in this case $\beta_r^{S_p} = N^n/\left[M_{klp}\ R_i^{(P)}\ C\right]$ is not independent of i for n > 2. Similarly, we can see that T_6-class is also empty for Midzuno scheme of sampling with or without replacement for any value of n > 1 and for Horvitz-Thompson's sampling scheme. The estimator for such schemes jumps to the new class as observed in Section 5.1.

For SRSWR and n=2, the classical estimator lies in T_6-class and also in a different new class by Tikkiwal [11], which depends on an S_p only, for all $s_t \in S_p$. These observations are noted as a special case of the general discussion in the following section.

5.3 Emptiness or otherwise of T_6-class for sampling with varying probabilities with replacement.

For simplicity we limit our discussions to the case for the sample size n = 2. In this case the set S_p consists of samples containing either one distnict unit or two distnict units. Thus , M_{k1p}, the number of samples in an S_p is either one or two. If the probability of selecting i-th unit at r-th draw is p_i for all i and r = 1, 2 such that Σ_i (pi) = 1; then

$$\beta_r^{S_p} = \begin{cases} \dfrac{1}{2Np_i^2} & \text{, for an } S_p \text{ consisting of samples with i–th unit alone as distinct units ;} \\ \dfrac{1}{2Np_ip_j} & \text{, for an } S_p \text{ consisting of samples with ith and j–th units as distinct units.} \end{cases}$$

for r = 1, 2 and all S_p. In this case the $\beta_r^{S_p}$ is not only independent of i but also of r and so depends only on S_p. Thus, its properties are perhaps better discussed by treating this estimator in the different new class discussed in the previous section. Otherwise proceeding as in Section 3, we can prove, using (5.1.5), that $\beta_r^{S_p} = 0$ for all r and S_p there by showing that MVLUE does not exist in T_6-class for this sampling scheme.

6.On the Non-existence of MVLUE in T_7-Class

An estimator based on the sample s_t in T_7-class is given by

$$\hat{T}_7 = \sum_{r=1}^{n} \left(\beta_{r0}^{s_t} X_r \right) \tag{6.1}$$

The total number of weights $\beta_{r0}^{s_t}$ is nN^n in general. These weights are fixed in advance. The condition of unbiasedness gives rise to

$$\Sigma_{(1)} \left[\Sigma_r \beta_{r0}^{s_t} \delta_{ri} \right] p(s_t) = 1 \tag{6.2}$$

for i = 1, 2, . . . , N. With

$$\delta_{ri} = \begin{cases} 1, & \text{if i–th unit occurs at the r–th draw in a given } s_t \in S', \\ & \text{the sample space of all possible samples} \\ 0, & \text{otherwise} \end{cases}$$

A consistant solution of these N equations is given by

$$\beta^{s_t}_{r_o} = \frac{p_{ir}}{\sum_r (p_{ir}\, \delta_{ri})\, p\,(s_t)\, C} \tag{6.3}$$

for $t = 1, 2, \ldots, N^n$, $r = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, N$ with

$C = \left[\sum_{k=1}^{n} \sum_{l=1}^{\binom{N-1}{k-1}} \sum_{p=1}^{M_{kl}} M_{klp} \right]$ and p_{ir} denoting the probability of selecting i-th unit at r-th draw. Hence T_6-class is non-empty.The variance of the estimator is given by

$$V(\hat{T}_7) = \Sigma_{(1)} \left[\left(\Sigma_r\, \beta^{S_p}_{ro}\, X_r\right)^2 \; p(s_t) \right] - T^2 \tag{6.4}$$

After differentiating the function

$$\emptyset = V(\hat{T}_7) - 2 \sum_i \lambda_i \left[\Sigma_{(i)} \left(\left(\Sigma_r\, \beta^{S_p}_{ro}\, \delta_{ri} \right) p(s_t) \right) - 1 \right] \tag{6.5}$$

with respect to $\beta^{s_t}_{ro}$ and equating $\delta\emptyset/\delta\beta^{s_t}_{ro}$ to zero, we get

$$\left[\Sigma_r \left(\beta^{s_t}_{ro}\, X_r \right) x_{i_r} \right] = \lambda_{i_r} \tag{6.6}$$

with X_{i_r} denoting the variate value of the unit that occurs at r-th draw for a given sample s_t. For population vector

$$X = [\, X_{i_r} = 1, \; x_{i'} = 0 \;\; \text{for} \;\; i' (\neq i_r) = 1, 2, \ldots, N],$$

(6.6) gives $\beta^{s_t}_{ro} = \lambda_{i_r}$ and for population vector $X = [\, x_i = 0$ for all $i \in s_t$ and arbitrary values for others] gives $\lambda_i = 0$. Thus $\beta^{s_t}_{ro} = 0$ for any given s_t, r and i. But this choice of $\beta^{s_t}_{ro}$ does not satisfy the conditions of unbiasedness, thus leading to a contradiction. Hence MVLUE does not exist in this class.

REFERENCES

[1] Basu, D., 1958. On sampling with and without replacement, *Sankhya*, **20**, 287-294.

[2] Bhargava, N.K. and Tikkiwal, B.D, 1978. On the axioms of sample formation with their relevance to T-Classes of linear estimators, *Sankhya Series C*, **40**, 9-18.

[3] Godambe, V.P., 1955. A unified theory of sampling from finite population, *Jour. Roy. Statist. Soc.,B*,17, 269-278.

[4] Horvitz, D.G. and Thompson, D.J., 1952. A generalisation of sampling without replacement from a finite universe. *Jour. Amer. Statist. Assosc.*, **47**, 663-685.

[5] Koop, J.C., 1961. Contributions to the general theory of sampling finite population with replacement and with unequal probabilities. *Mimeo Series No. 296, Inst. of Statistics, North Carolina (Ph.D. Thesis, 1957, N.C. State College).*

[6] ________, 1963. On the axioms of sample formation and their bearing on the construction of linear estimators in sampling theory for finite population. *Metrika 7 (2 and 3)* **8**-114, 165-204.

[7] ________, 1978. Comments on the paper by Bhargava and Tikkiwal, *Sankhya*, C, **40(1)** : 19-20.

[8] Prabhu-Ajgaonkar, S.G. and Tikkiwal, B.D., 1961. On Horvitz and Thompson's T-Classes estimators, *Ann. Math. Statist.*, **32**, *923 (Abstract).*

[9] Tikkiwal, B.D., 1972. On unordering of estimators in sampling with or without replacement and its impact on T-Classes of estimators, *Abstract, Inst. Math. Stat. Bulletin.*

[10] Tikkiwal, B.D. and Tikkiwal, G.C., 1979. Theoretical foundations of inference from finite population and data analysis. *Proceedings of International Conference in Statistics at Tokyo*, 85-88.

[11] Tikkiwal, G.C., 1988. A note on T-classes of linear estimators, *Ganita Sandesh*, **2**,1, 32-38.